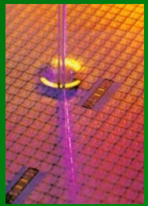


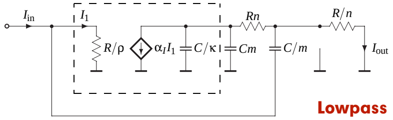
Fundamental Frequency Limitations in Current-Mode Sallen-Key Filters

Hanspeter Schmid and George S. Moschytz, Signal and Information Processing Lab., ETH Zürich, Switzerland. hanspeter.schmid@isi.ee.ethz.ch

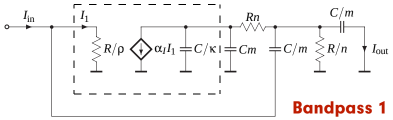
Summary

We investigated high-frequency current amplifier non-idealities and their effects on all-pole Sallen-Key filter biquads. It can be shown that current amplifier non-idealities cause parasitic zeros in the filter transfer functions, and thus impose fundamental limitations on the realizable pole frequency. Design equations are given, providing compensation for the amplifier's port impedances and phase lag, by predistortion of the component values. It is also shown how design specifications for a current amplifier can be derived from the filter specification, minimizing the amplifier's power dissipation. Finally, a design procedure is discussed by the example of a video-frequency lowpass filter biquad.

Current-Mode Sallen-Key Filters



Lowpass



Bandpass 1

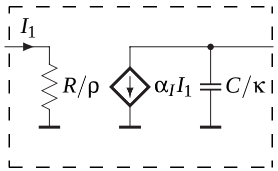
Both a **Bandpass 2** and a **Highpass** filter are easily derived by exchanging Rs and Cs in the Bandpass 1 and Lowpass filter, respectively. Then, assuming an ideal amplifier,

$$\left(\omega_{\mathrm{p\,i}}, \frac{1}{q_{\mathrm{p\,i}}}\right) = \left(\frac{1}{RC}, \frac{m^2n^2+m^2+(\alpha_I+1)}{mn}\right), \quad (2_{\mathrm{LP}})$$

$$\left(\frac{1}{\sqrt{2}\,RC}, \frac{m^2n^2+m^2+(\alpha_I+2)}{\sqrt{2}\,mn}\right), \quad (2_{\mathrm{BP1}})$$

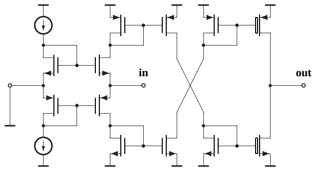
with similar expressions for BP2 and HP.

Current Amplifier Model



ρ and κ describe the port impedances in relation to the passive part's R and C. Note: $\alpha_I < 0$ in Sallen-Key filters, because the VCVS and CCCS definitions are *not* dual.

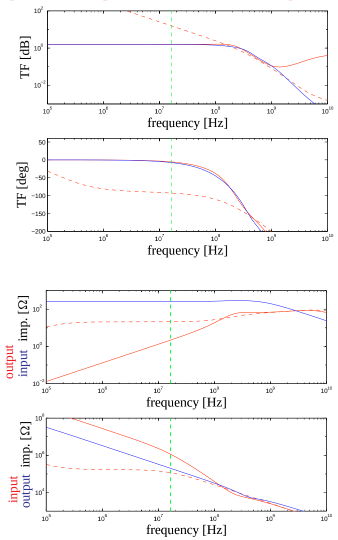
Open-Loop Current Amplifier



This schematic shows the current amplifier used for the design example (it is also called *second generation current conveyor, CCII*). The transistor symbols with the "boxed" gates indicate super transistors. We used regulated cascodes in our example. The simulated values at 16.5 MHz:

$\alpha_I = -1.57$, $C_{\mathrm{out}} = 0.048\,\mathrm{pF}$, $R_{\mathrm{in}} = 254\,\Omega$, $\phi = 7.65°$

Open-Loop vs Closed-Loop Amp



These figures show the transfer function and the input and output impedances of

blue: A CCII used as a current buffer

red: An opamp (AD SSM 2135) connected as a buffer

red dashed: dito, open-loop

Note: the opamp is actually 50 times slower, the red curves are scaled in frequency (×50) and magnitude (×1.57).

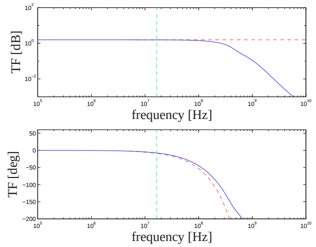
Towards the frequency limits, the advantages of the closed-loop amplifiers vanish together with the loop gain.

Phase Model for the Amplifier

The current amplifier does not have one *dominant* pole. It can be seen that a model in which the phase lag is linearized through the points at $\omega = 0$ and $\omega = \omega_{\mathrm{p}}$ results in a better fit of model and "reality" than a one-pole model. Thus

$$\alpha_I(s) = \alpha_{I\,\mathrm{DC}} \cdot e^{\left(-\phi\,\frac{s}{\omega_{\mathrm{p}}}\right)},$$

where ϕ is the pase lag at the filter's pole frequency ω_{p}. The figure below shows the simulated and the analytically modeled transfer function of the buffer. Green: ω_{p}.



Analytical treatment

The filter transfer functions can then be calculated using the buffer model with input resistance, output capacitance and linear phase lag.

Since the analytical phase model is accurate only close to the pole frequency, it applies neither to higher-frequency poles nor to parasitic zeros. Therefore numerator and denominator of the transfer functions must be treated seperately: in the numerator, ϕ is set to zero. The non-linear function in the denominator is approximated by a Taylor series, neglecting the cubic and higher-order terms.

The amplifier non-idealities shift the poles of e.g. the lowpass filter to

$$\frac{\omega_{\mathrm{p}}^2}{\omega_{\mathrm{p\,i}}^2} = \frac{\rho\kappa mn}{\rho\kappa(mn-\phi\alpha_I)+\rho n+(\kappa m+1)(n^2+1)}, \quad (3_{\mathrm{LP}})$$

$$\frac{1}{q_{\mathrm{p}}} = \frac{\rho\kappa(m^2n^2+m^2+(\alpha_I+1))+\rho m(n^2+1)+\kappa n}{\sqrt{\rho\kappa mn}\sqrt{\rho\kappa(mn-\phi\alpha_I)+\rho n+(\kappa m+1)(n^2+1)}}, \quad (4_{\mathrm{LP}})$$

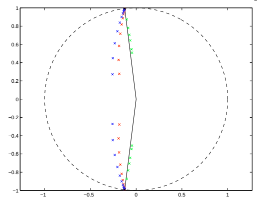
and, more importantly, cause one or two zeros (and also, for the BP1, a third pole at $\omega_{\mathrm{p}}' = -\alpha_I \cdot \omega_{\mathrm{z}}^2/\omega_{\mathrm{p}}$):

$$(\omega_{\mathrm{z}}^2, q_{\mathrm{z}}) = \left(\frac{1}{R^2C^2}\,\frac{-\rho\alpha_I m}{n\left(m+\frac{1}{\kappa}\right)}, \sqrt{-\rho\left(m+\frac{1}{\kappa}\right)\alpha_I mn}\right), \quad (5_{\mathrm{LP,\,BP1}})$$

$$\omega_{\mathrm{z}} = \frac{1}{RC}\,\frac{1}{mn+\frac{n}{\kappa}-\rho\alpha_I m}. \quad (5_{\mathrm{HP,\,BP2}})$$

Pole Shift (Lowpass Filter)

This figure shows the poles in the normalized s-plane. The circle is the location of constant frequencies ($\omega_{\mathrm{p}} = 1$), the lines are the locations of constant pole quality ($q_{\mathrm{p}} = 4$).



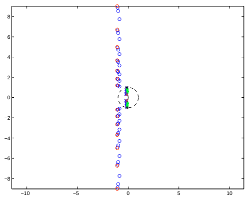
blue: $\kappa = 1.0, 1.8, 3.2, 5.6, 10, 18, 32, 56, 100, \ldots$

red: $\rho = 1.0, 1.8, 3.2, 5.6, 10, 18, 32, 56, 100, \ldots$

green: $\phi = 3, 6, 9, 12, 18, 21°$

Parasitic Zeros (Lowpass Filter)

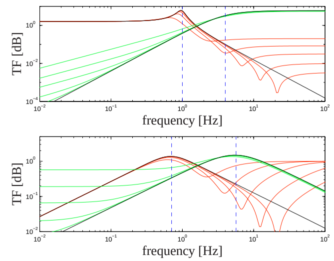
The following figure is the same as above, in a larger scale, showing the parasitic zeros for some values of ρ and κ.



red: $\rho = 1.0, 1.8, 3.2, 5.6, 10.0, 17.8, 31.6$, $\kappa \to \infty$

blue: ρ as above, $\kappa = 10, 33, 100$

Transfer Functions ($\kappa = 30$, $\rho = 3\ldots300$)



These plots show the normalized transfer functions of the Lowpass, Bandpass 1, Bandpass 2 and Highpass filters for constant κ and different values of ρ.

Frequency Limitations (LP)

The maximum possible ω_{p} depends on the component spread and the stopband attenuation:

$$\omega_{\mathrm{p\,max}} \approx \frac{1}{10\cdot\max(m,1/m)\,C_{\mathrm{o}}\cdot\max(n,1/n)\,R_{\mathrm{i}}\cdot A_{\mathrm{stop}}}.$$

Design: $$C_{\mathrm{o}}R_{\mathrm{i}} \le \frac{1}{\omega_{\mathrm{p}}}\cdot\frac{1}{10\cdot\max(m,1/m)\cdot\max(n,1/n)\cdot A_{\mathrm{stop}}}.$$

Design Procedure and Example

$f_{\mathrm{p}} = 16.5\,\mathrm{MHz}$, $q_{\mathrm{p}} = 4$, $A_{\mathrm{stop}} = 30\,\mathrm{dB}$, $m = 0.6$, $n = 1.$

(1) blue: $C \approx 20\cdot C_{\mathrm{out}} = 0.96\,\mathrm{pF}$, $R = 10.04\,\mathrm{k}\Omega$.

(2) cyan dashed: R corrected by (3_{LP}): $R = 8.20\,\mathrm{k}\Omega$.

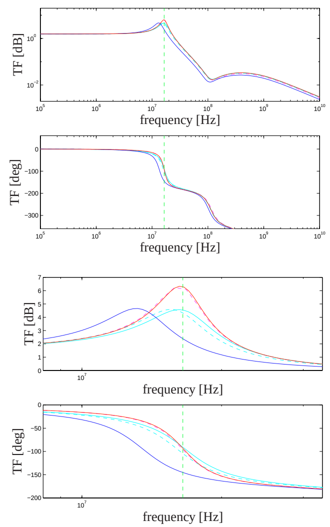
(3) cyan: $1/R$ linearly extrapolated: $R = 7.81\,\mathrm{k}\Omega$.

(4) magenta dashed: m corrected by (4_{LP}): $m = 0.586$.

(5) red: m linearly extrapolated: $m = 0.566$.

Simulated values (BSIM 3v3, 0.6 µm CMOS process):

#	f_{p} [MHz]	q_{p}	Δf_{p}	Δq_{p}
(1)	13.39	2.98	−19%	−26%
(2)	15.83	2.91	−4%	−27%
(3)	16.46	2.91	≈ 0%	−27%
(4)	16.40	3.95	−0.5%	−1.3%
(5)	16.40	4.01	−0.5%	≈ 0%



Acknowledgements

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