

# Efficient simulation of harmonic distortion

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$$\text{IM3} = \frac{3(y_1 - 2y_2 + 2y_4 - y_5)}{2(4y_1 - 5y_2 + 5y_4 - 4y_5)}$$

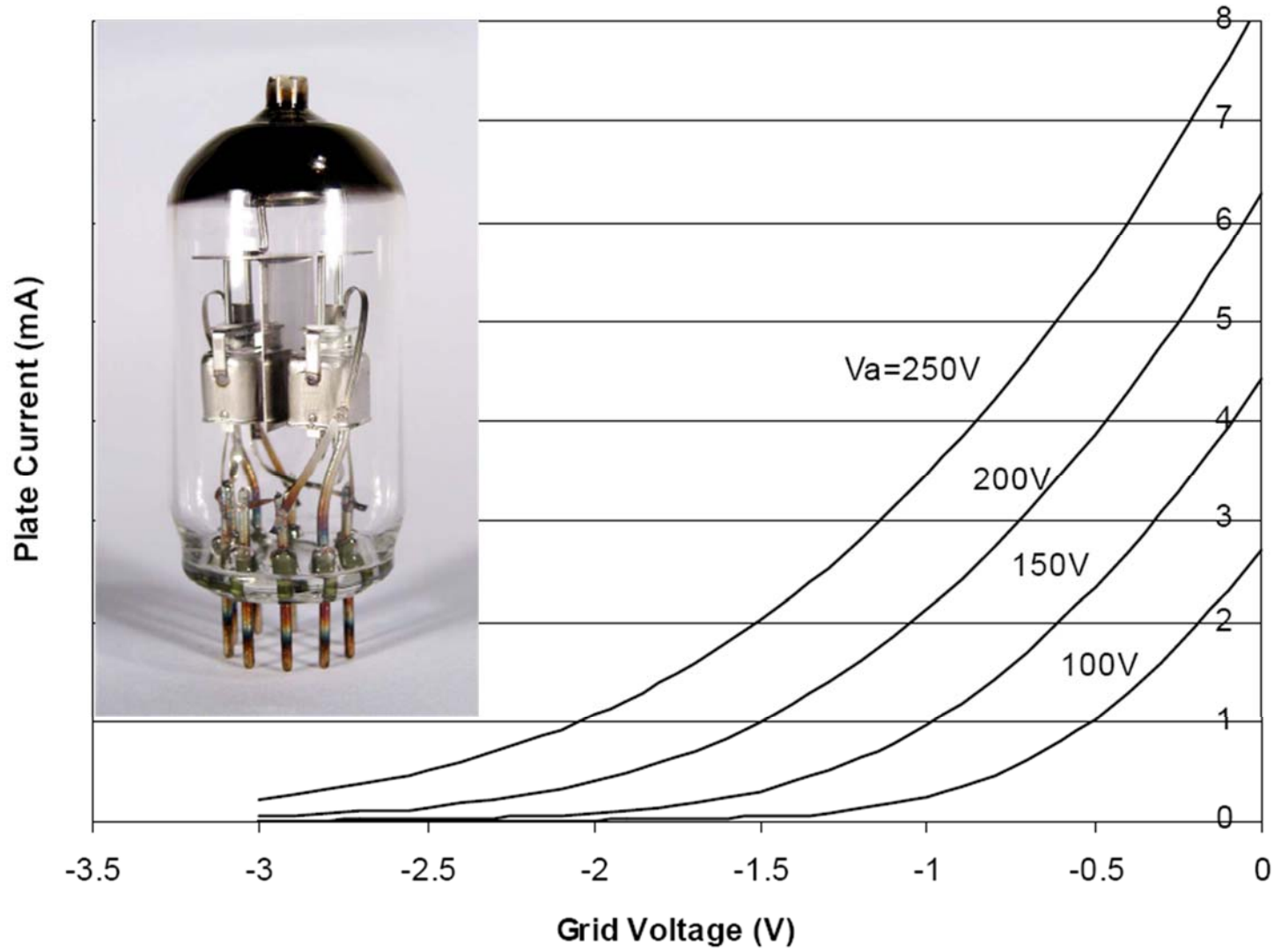
## Using equidistant points on a transfer characteristic to calculate harmonic distortion

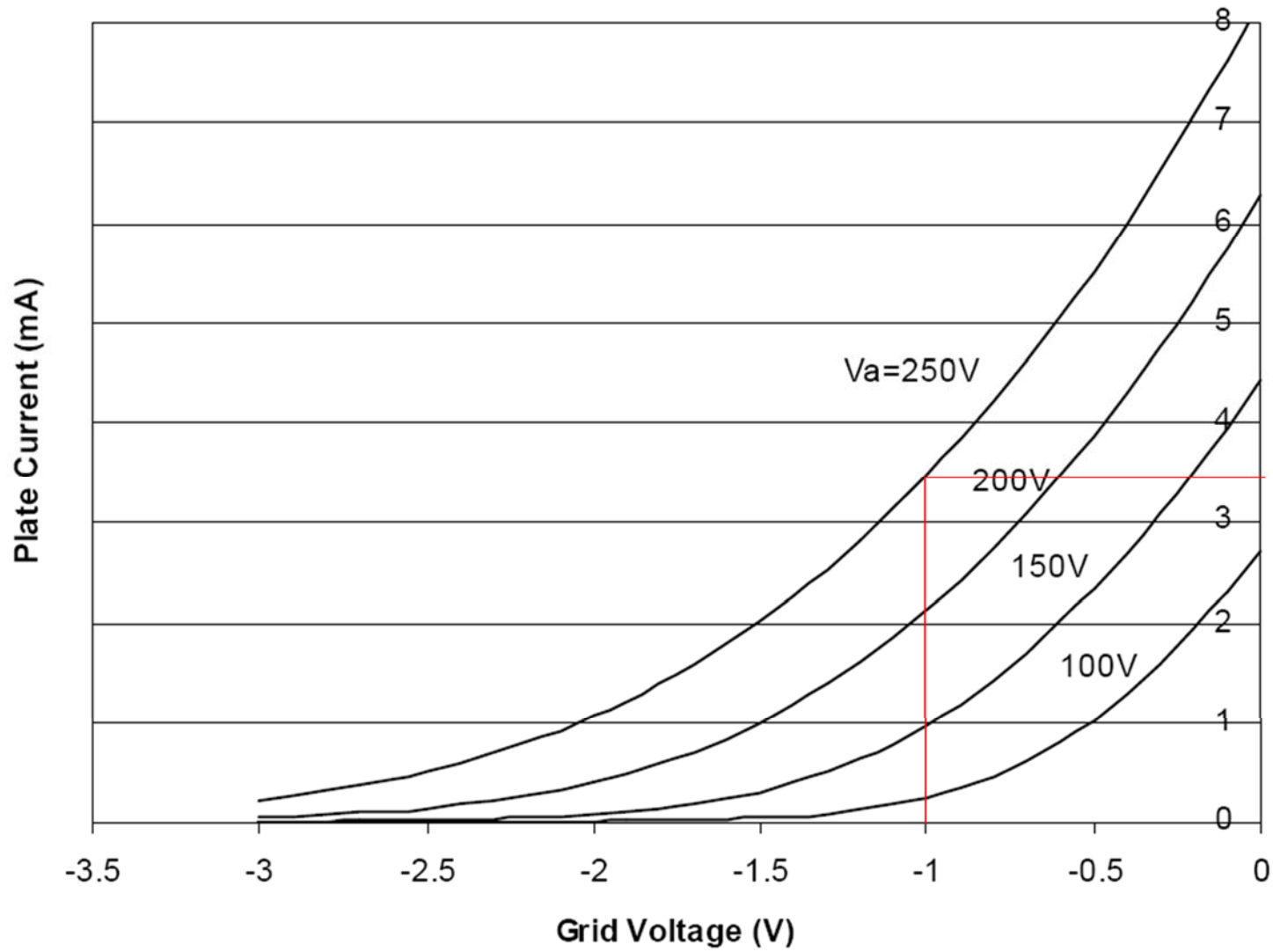
- How can you determine  $H_2$  with pen and paper?  
(Espley 1933)
- What can be calculated with  $N$  equidistant points?
- What are the underlying mathematical principles?
- Which previously open questions does my paper close?
- How can it be applied to speed up simulations?  
(Discrete-time analogue circuits)
- What will you find in the paper?
- Conclusion

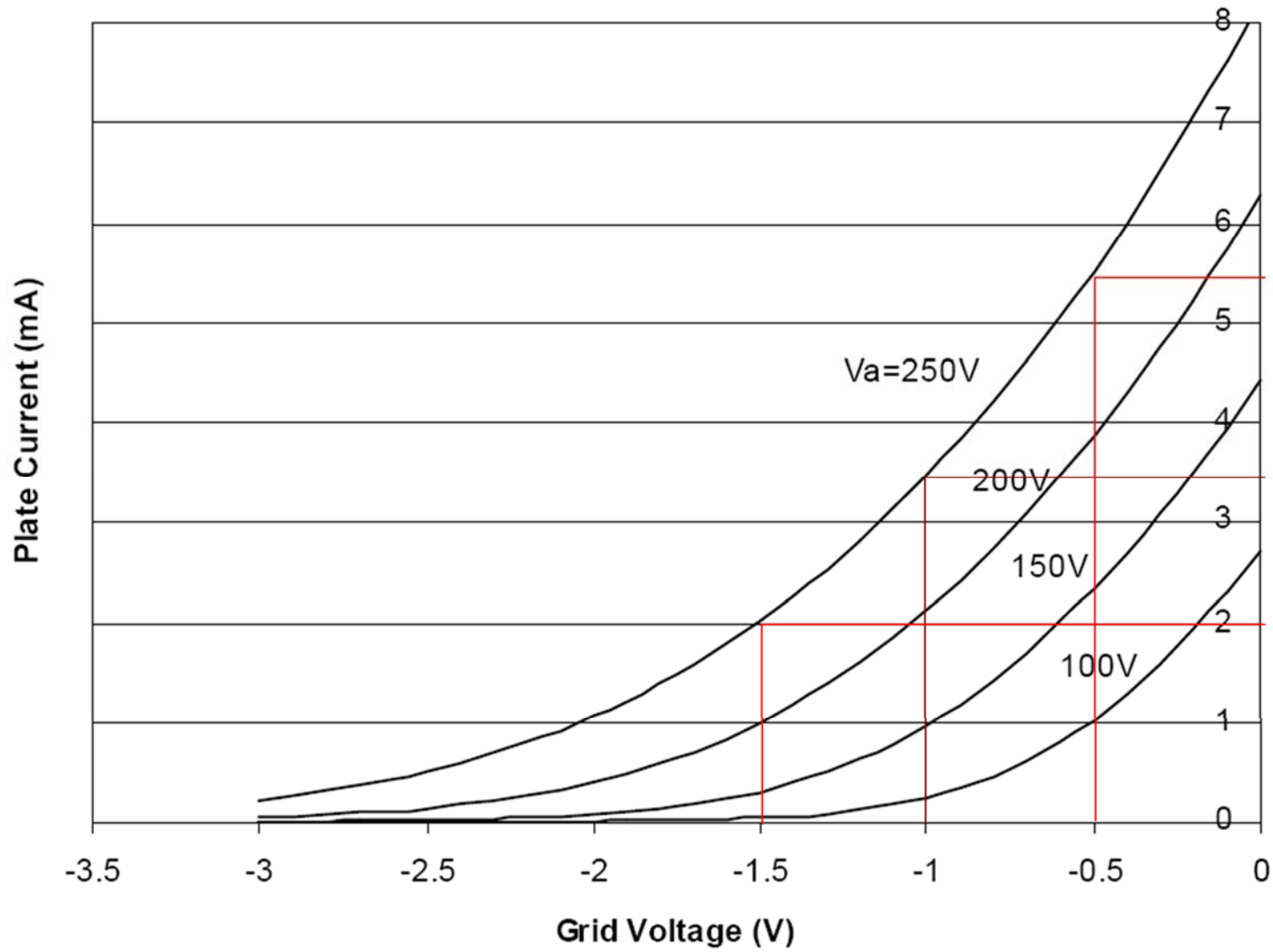
**n** | *w*

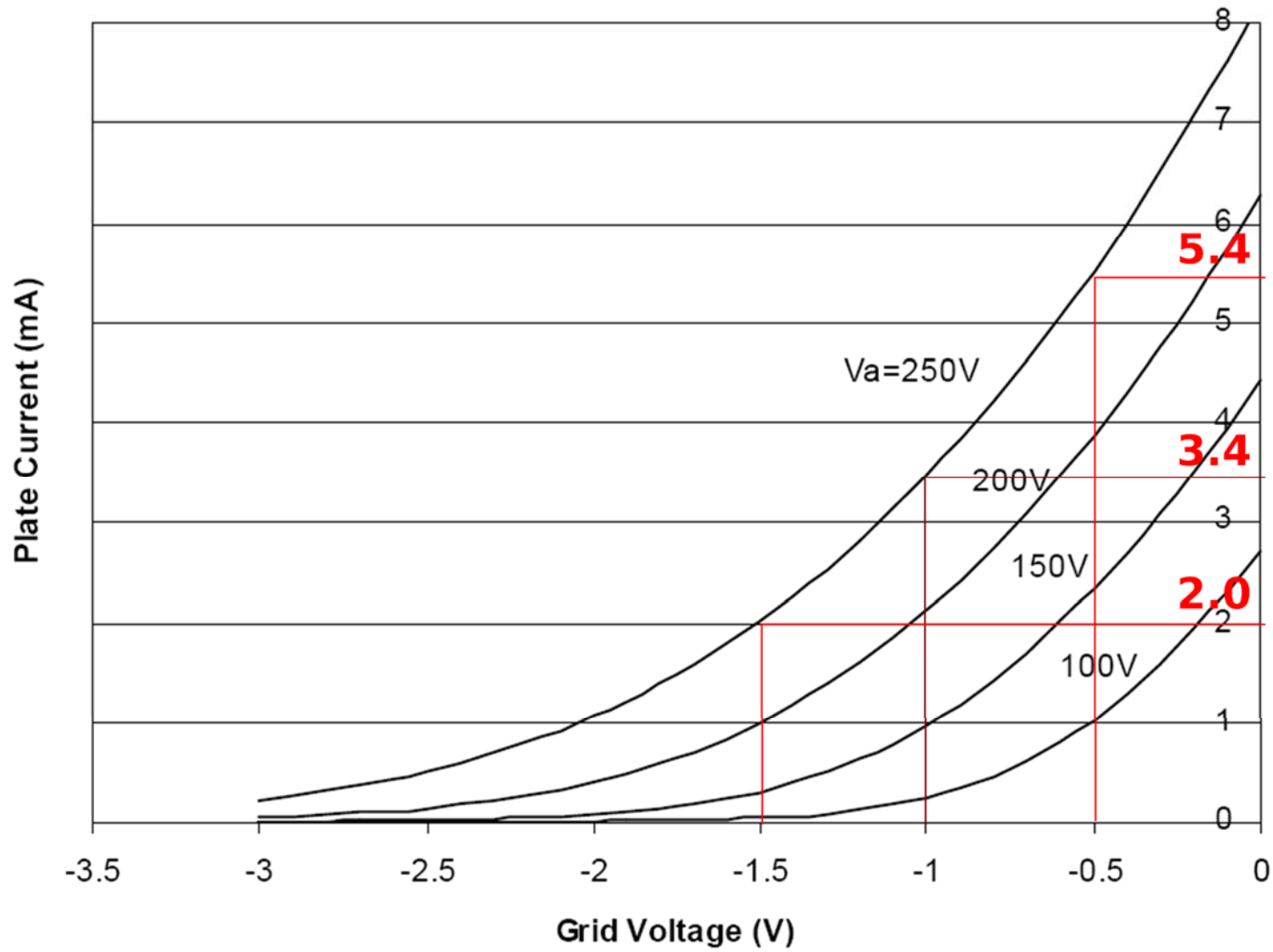


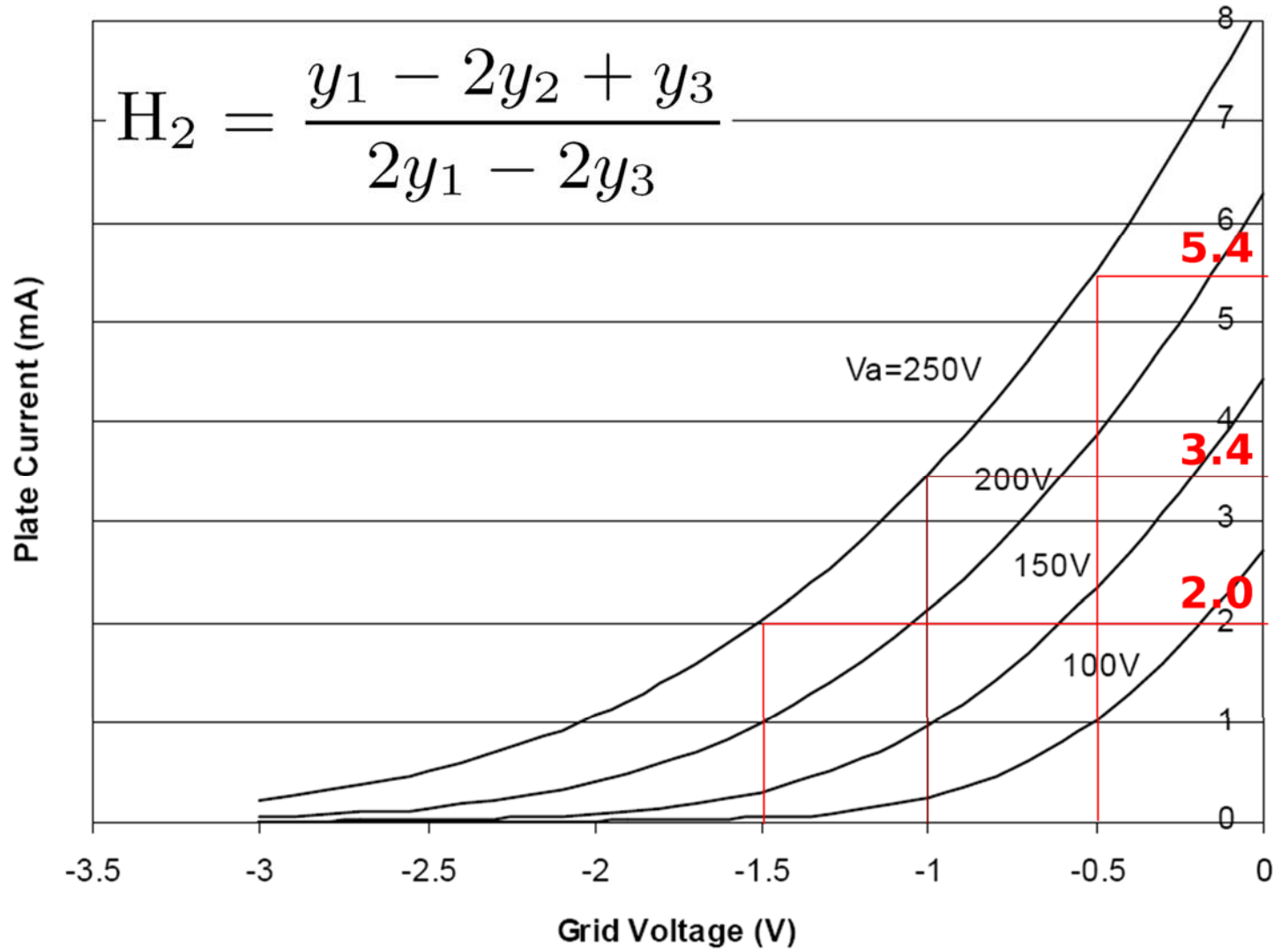
## **HD with pen and paper: Espley 1933**

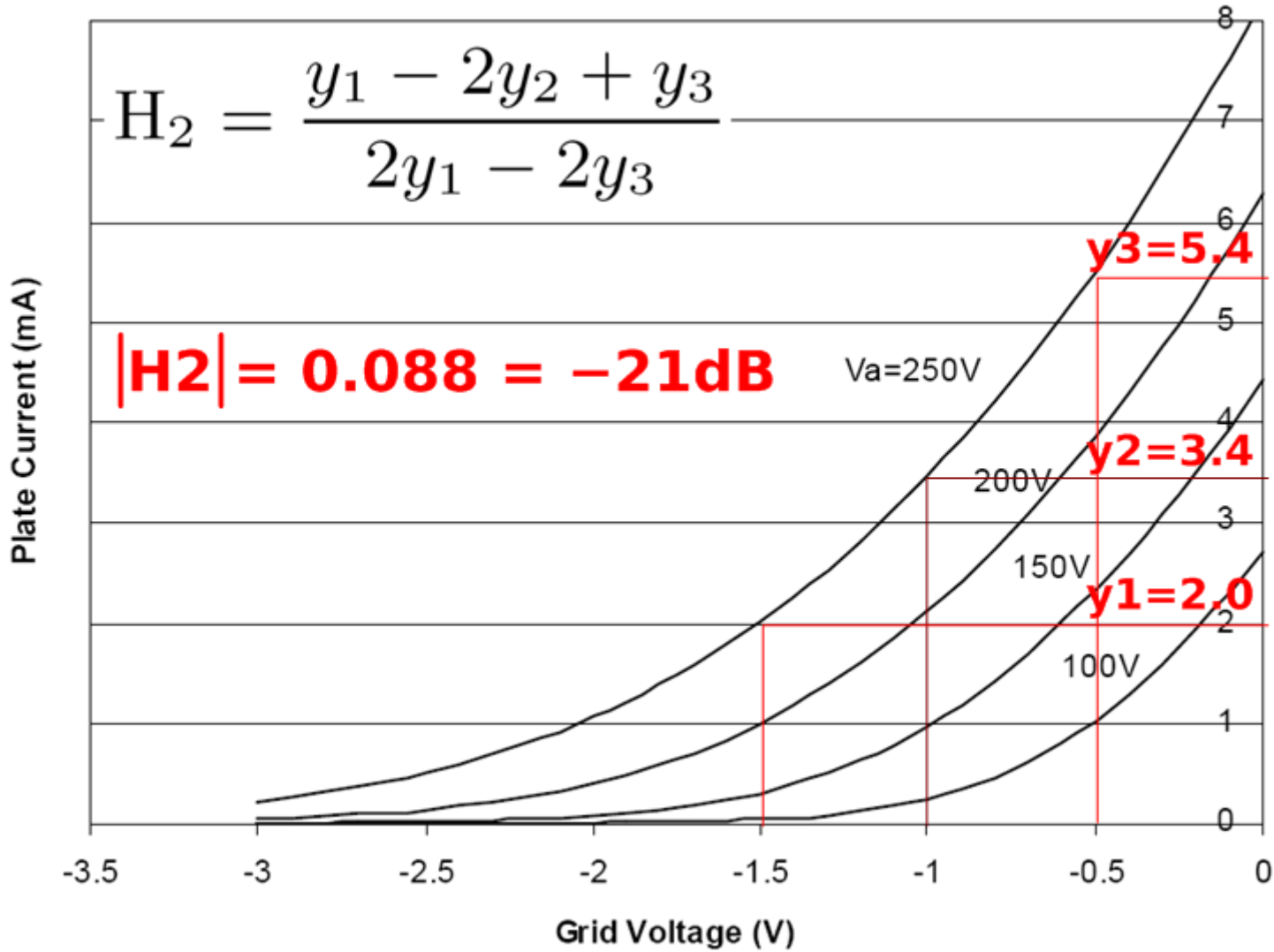








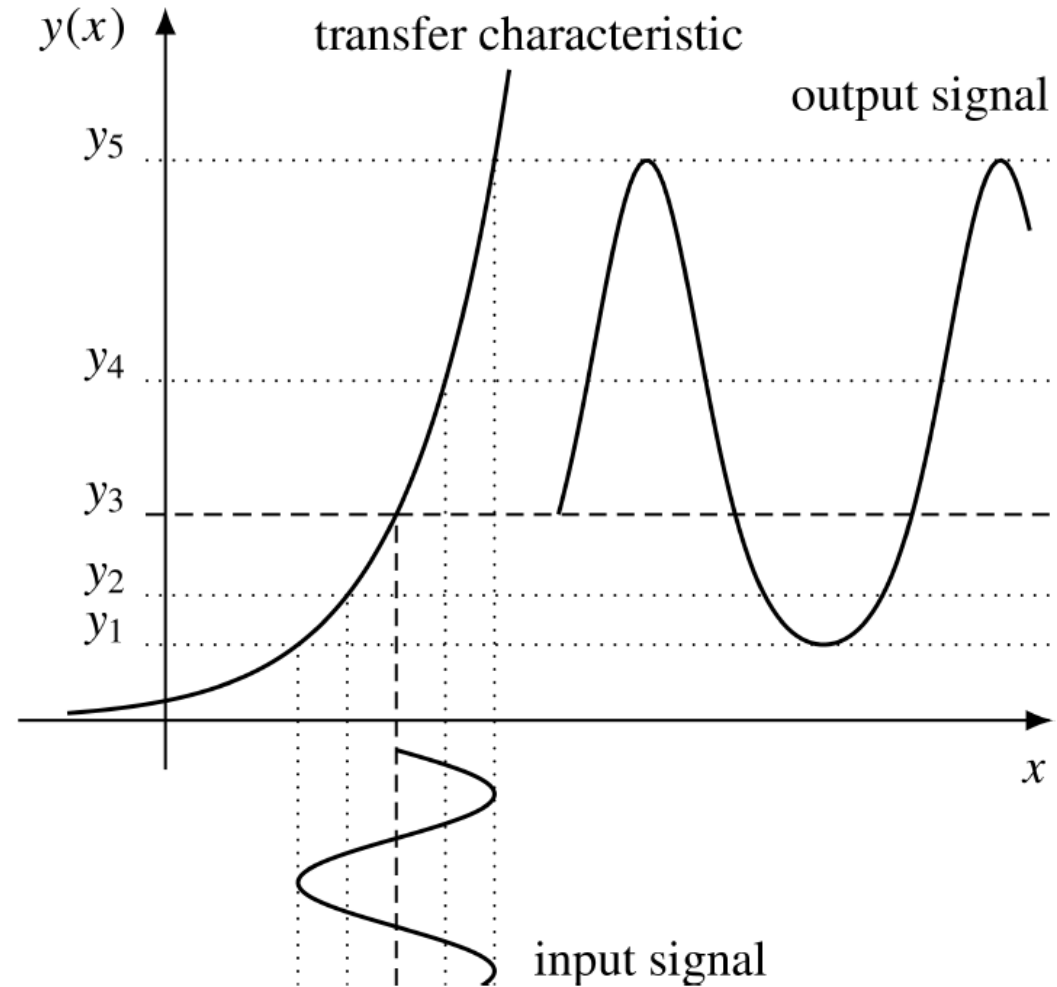




Can be generalized to N points, e.g., N=5

N=5 lets us calculate:

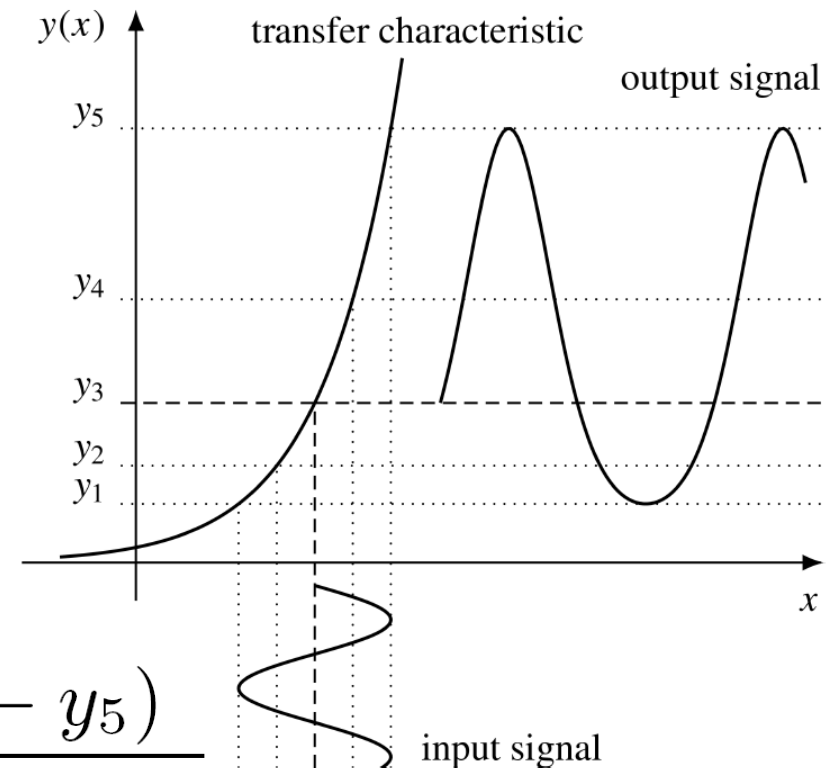
- 2<sup>nd</sup>-order distortion
- 3<sup>rd</sup>-order distortion
- 4<sup>th</sup>-order distortion
  
- 3<sup>rd</sup>-order intermodulation distortion
  
- distortion-induced offset



## Mathematical principles

Instead of calculating the distortion for the characteristic itself, we

- choose N equidistant points on it,
- lay a polynomial through these points,
- and calculate the distortion that would be caused if that polynomial were the characteristic.



$$\text{IM3} = \frac{3(y_1 - 2y_2 + 2y_4 - y_5)}{2(4y_1 - 5y_2 + 5y_4 - 4y_5)}$$

## What has been done with this method

- Espley: graphical determination of HD from a few equidistant points on the transfer characteristic (1933/34)
- Medhurst: estimation of the transfer characteristic from a few measured distortion values (1966)
- Bruun: using Espley's formulae to obtain simple analytic approximations for mismatch-induced distortion (1999)
- Schmid: speeding up distortion simulations (2000)
- Giustolisi, Palumbo: calculating distortion in class-AB stages (2003)

**Open questions ... at least until today:**

What is the error of using an equidistant-point approximation?

Minimum number of points needed for n-th order distortion?

Would non-equidistant points give better results?

Is it better than using a discrete Fourier transform (DFT)?

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Less than 0.1 dB for harmonic distortions (HDs) of  $< -30$  dB.

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The first odd number greater than n, for HDs  $< -30$  dB.

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Not for HDs  $< -30$  dB.

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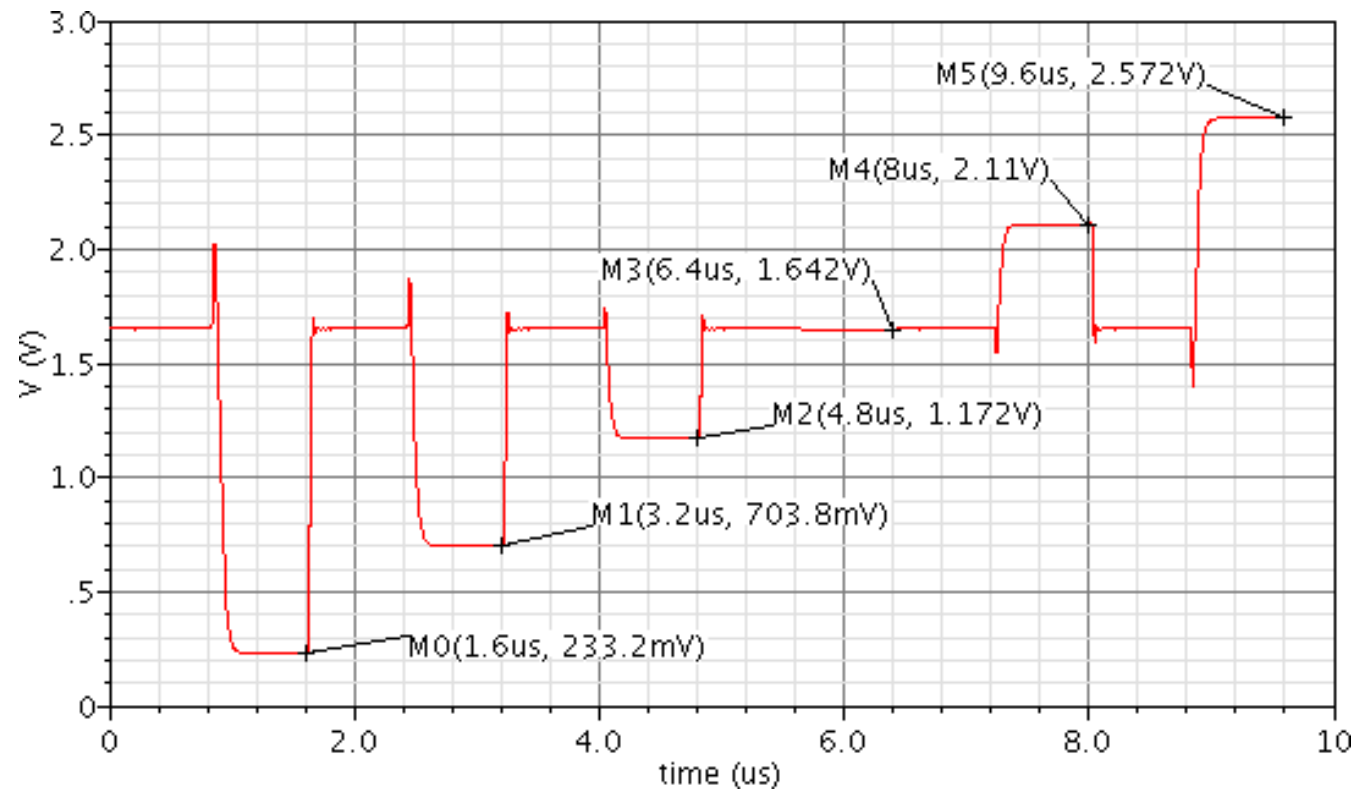
Is it better than using a discrete Fourier transform (DFT)?

As a rule of thumb, a  $2(n - 1)$ -point-DFT and an n-equidistant-point calculation give similar errors.

	DFT	Espley	Espley	Espley	Espley	DFT	DFT	DFT
	1024	3	5	7	9	8	10	12
<b>Transfer Characteristic T1</b>								
HD3	-11.315	---	-12.523	-10.415	-12.412	-15.468	-12.878	-11.988
HD5	-18.826	---	---	-16.537	-20.281	---	---	-23.935
HD7	-25.698	---	---	---	-21.515	---	---	-23.935
IM3	-10.514	---	4.573					
<b>Transfer Characteristic T2</b>								
HD3	-20.042	---	-20.155	-19.966	-20.064	-21.204	-20.183	-20.060
HD5	-37.983	---	---	-37.145	-38.073	---	---	-39.185
HD7	-55.749	---	---	---	-54.141	---	---	-39.185
IM3	-15.132	---	-2.881					
<b>Transfer Characteristic T3</b>								
Off.	-3.634	-3.56	-3.636	-3.634	-3.634	3.634	3.634	3.634
HD2	-39.799	-39.706	-39.797	-39.799	-39.799	-39.796	-39.799	-39.799
HD3	-39.696	---	-39.697	-39.696	-39.696	-39.805	-39.698	-39.696
HD4	-73.357	---	-73.202	-73.358	-73.357	-67.336	-73.204	-73.354
HD5	-77.808	---	---	-77.724	-77.809	---	---	-77.918
HD6	-108.352	---	---	-108.099	-108.356	---	-73.204	-102.331
HD7	-115.862	---	---	---	-115.7	---	---	-77.918
HD8	-144.097	---	---	---	-143.761	---	---	-73.354
IM3	-30.827	---	-29.598					
<b>Transfer Characteristic T4</b>								
HD3	-59.954	---	-59.954	-59.954	-59.954	-59.965	-59.954	-59.954
HD5	-118.32	---	---	-118.312	-118.32	---	---	-118.331
HD7	-176.583	---	---	---	-176.567	---	---	-118.331
IM3	-50.481	---	-50.359					
<b>Transfer Characteristic T5</b>								
Off.	266.066	271.54	266.111	266.066	266.066	266.066	266.066	266.066
HD2	-12.389	-12.726	-12.392	-12.389	-12.389	-12.387	-12.389	-12.389
HD3	-28.129	---	-28.133	-28.128	-28.129	-28.023	-28.129	-28.129
HD4	-46.298	---	-46.373	-46.298	-46.298	-40.277	-46.369	-46.297
HD5	-66.369	---	---	-66.409	-66.369	---	---	-66.318
HD6	-88.004	---	---	-88.073	-88.004	---	-46.369	-81.984
HD7	-110.965	---	---	---	-111.01	---	---	-66.318
HD8	-135.078	---	---	---	-135.138	---	---	-46.297
IM3	-19.394	---	-20.425					
<b>Transfer Characteristic T6</b>								
Off.	-69.336	-71.921	-69.46	-69.345	-69.337	69.343	69.336	69.336
HD2	-17.46	-17.659	-17.454	-17.459	-17.459	-17.445	-17.46	-17.460
HD3	-32.42	---	-32.429	-32.412	-32.419	-32.054	-32.44	-32.419
HD4	-46.358	---	-46.77	-46.366	-46.358	-40.338	-46.784	-46.336
HD5	-59.735	---	---	-60.087	-59.749	---	---	-59.301
HD6	-72.758	---	---	-73.58	-72.809	---	-46.784	-66.737
HD7	-85.536	---	---	---	-86.273	---	---	-59.301
HD8	-98.134	---	---	---	-99.375	---	---	-46.336
IM3	-16.053	---	-24.051					

## Application in simulation I: shorter simulation time

SC-Amplifier with correlated double sampling and digital controller



## Application in simulation I: shorter simulation time

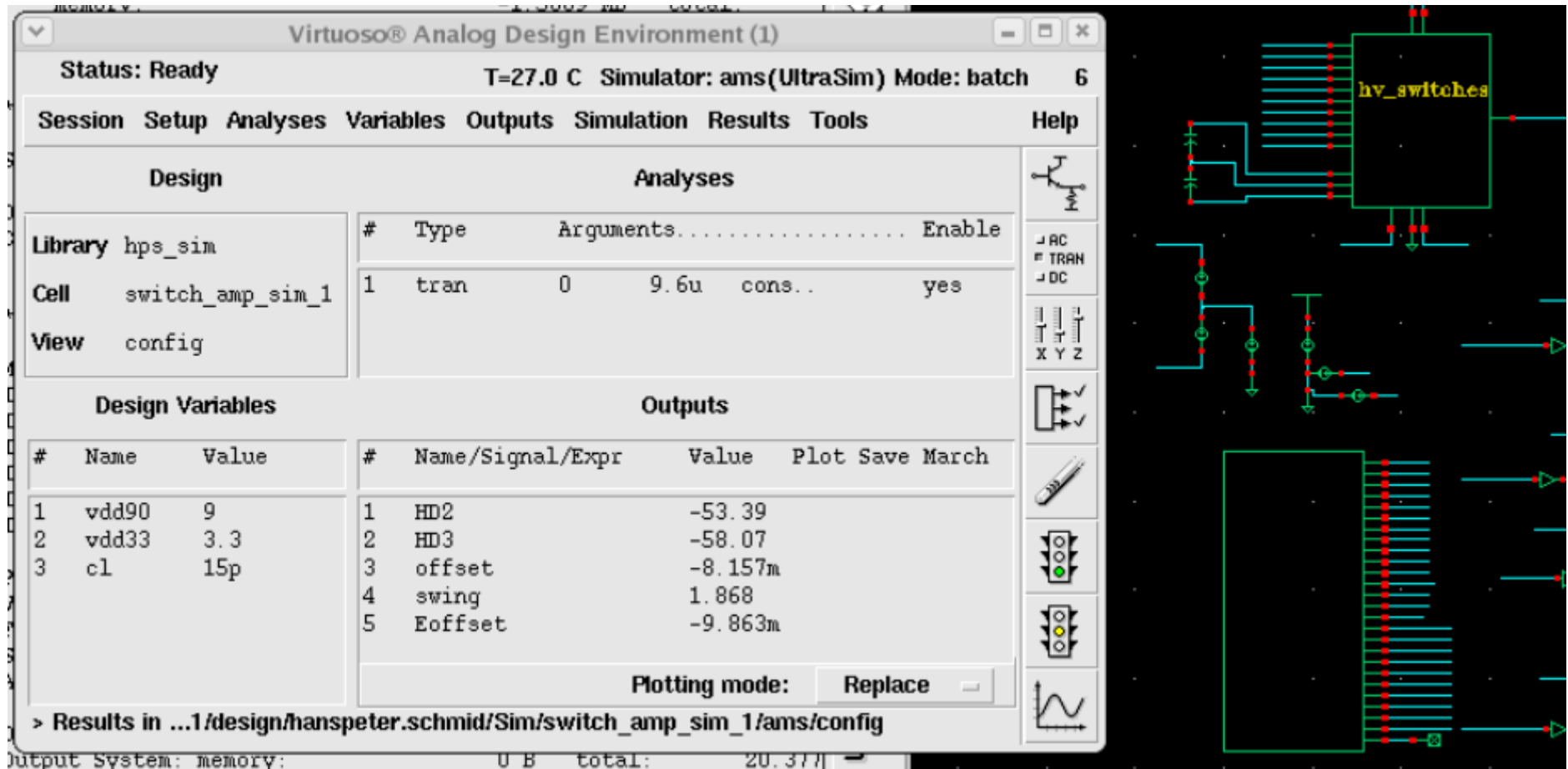
SC-Amplifier with correlated double sampling and digital controller

$$H_2 = \frac{3(y_1 - 2y_3 + y_5)}{4(y_1 + y_2 - y_4 - y_5)}$$

```
H2 = (20.0 * log10(abs((( (-3.0 / 4.0) *
  ((value(VT("/vout") 9.6e-06) - (2.0 * value(
    VT("/vout") 6.4e-06))) + value(VT("/vout")
    3.2e-06))) / ((value(VT("/vout") 9.6e-06) +
  value(VT("/vout") 8e-06)) - value(VT("/vout")
    4.8e-06) - value(VT("/vout") 3.2e-06))))))
```

## Application in simulation I: shorter simulation time

SC-Amplifier with correlated double sampling and digital controller



Virtuoso® Analog Design Environment (1)

Status: Ready T=27.0 C Simulator: ams(UltraSim) Mode: batch 6

Session Setup Analyses Variables Outputs Simulation Results Tools Help

Design Analyses

Library	hps_sim	#	Type	Arguments.....	Enable
Cell	switch_amp_sim_1	1	tran	0 9.6u cons..	yes

Design Variables

#	Name	Value
1	vdd90	9
2	vdd33	3.3
3	cl	15p

Outputs

#	Name/Signal/Expr	Value	Plot	Save	March
1	HD2	-53.39			
2	HD3	-58.07			
3	offset	-8.157m			
4	swing	1.868			
5	Eoffset	-9.863m			

Plotting mode: Replace

> Results in ...1/design/hanspeter.schmid/Sim/switch\_amp\_sim\_1/ams/config

Output System: memory: 0 B total: 20.377

## **Application in simulation I: shorter simulation time**

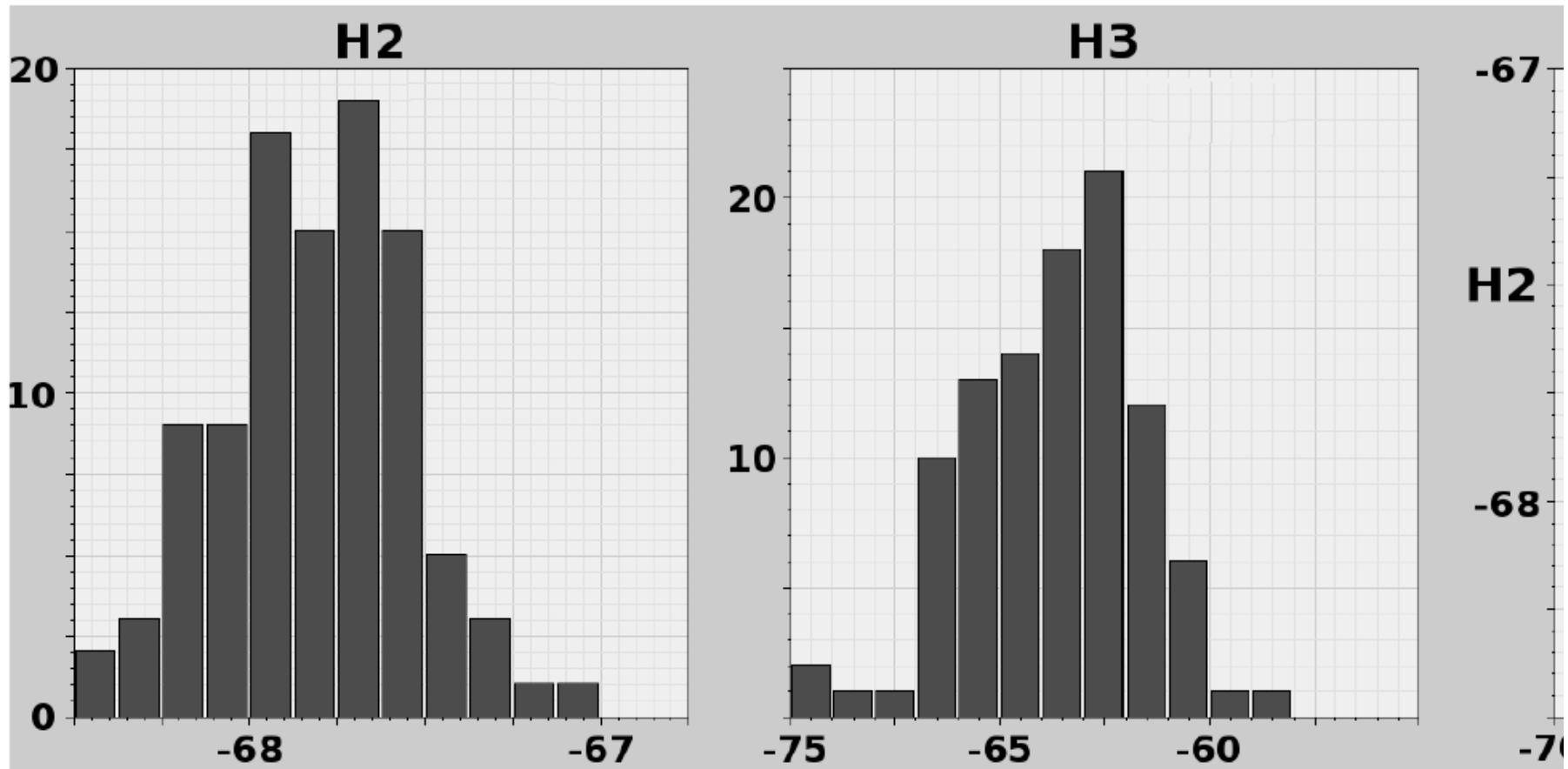
SC-Amplifier with correlated double sampling and digital controller

With equidistant points: 30 Minutes

With optimum (10-point) DFT: 55 Minutes

With recommended (64-point) DFT: 5.4 Hours

## Application in simulation II: Monte-Carlo Simulation



## What you will find in the paper

- The mathematical derivation of the fractions of weighted sums,
- a table of all fractions up to  $N=9$ ,
- a description of how I evaluated the approximation precision,
- an example of how to use the method for getting simple symbolic expressions for harmonic distortion,
- and, of course, references to literature.

**Conclusion: I showed you ...**

- a very old and simple method to calculate distortion
- that can be used to speed up simulations  
(or make Monte-Carlo simulations feasible)
- that I extended to distortion-induced offset
- and of which I determined the precision of approximation.

**Thank you!      Questions?**

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<http://www.schmid-werren.ch/hanspeter/>